

Graphs of quadratic equations

What you should know

How to complete the square on a quadratic expression.

How to use Venn diagrams.

New idea

The graph of $y = (x + a)^2 + b$ has a minimum at $(-a, b)$.

Task: Categorising quadratic curves

Think about all quadratic curves with equation $y = (x + a)^2 + b$.

Think also about these three properties.

A: The turning point has a positive x -value.

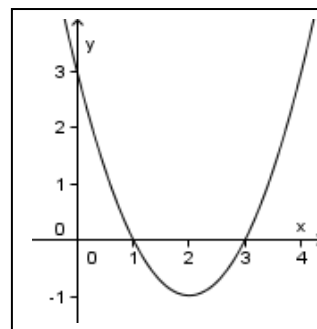
B: The turning point has a positive y -value.

C: The y -intercept is positive.

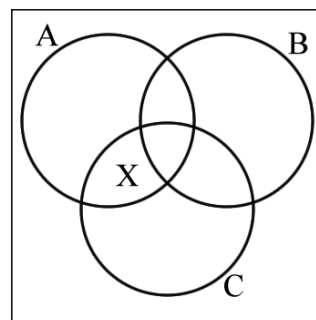
The quadratic curve shown on the right satisfies properties A and C but not B.

Its equation can be written as $y = (x - 2)^2 - 1$.

You could write this in the region marked with an X in the Venn diagram below.



- Copy the Venn diagram.
- Can you find a quadratic which doesn't satisfy any of the properties? Write this in the region outside the three circles.
- Can you find one equation for each of the other six regions?
- Is it possible to find an equation for every region?

**Take it further**

Find three other properties A, B and C for which all eight regions can be filled in.

Where this goes next

At A level you will learn more about the usefulness of completing the square in Core Mathematics. You will also learn another method for finding maximum and minimum points on cubics and other curves.